Euclid, Agrarian Arbitrator

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In the effort to establish congruence theorems as stock-in-trade for the pupils of the tenth grade, probably most teachers have vacillated between accepting the modern recommendation of informal demonstration and the old idea of building a logical structure from the outset by means of rigid proof. No one questions the difficulty of making children understand proof by "placing," especially in the early part of the first year of demonstrative geometry, but in spite of this, many teachers would like to conclude these theorems satisfactorily in their proper sequence.

In an attempt to do this and yet take into account the perplexities of the immature mind, the following sketch was worked out, seemingly successfully. The part of the defendant was played in a convincing way by a pupil who had completed a year of geometry. The other members of the cast and the jury were selected from the tenth grade class. The verdict brought in was entirely honest and was unanimously in favor of congruence.

After the court adjourned the class discussed the piece and decided to accept the general fact of congruence of triangles by two sides and the included angle. Better than that, impersonating lawyers, they experimented with triangles in which two angles and the included side of one were known to be equal to two angles and the included side of the other and succeeded in proving them congruent with none of the usual incredulity apparent. All this was accomplished in a class period of forty-two minutes.

Scene—Court-room with table and chair, gavel, etc., for judge, chairs for plaintiff and defendant, etc.

Clerk *enters, dressed in academic gown*

Clerk. Hear ye! Hear ye! This honorable court is now in session pursuant to adjournment. Govern yourselves accordingly. (*People stand while judge, dressed in academic gown, enters and takes his seat.*)

Judge. Mr. Clerk, what is the first case?

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CLERK. Vanya *vs.* Government.

JUDGE. Mr. Clerk, bring in a jury.

CLERK. The jury has been selected and is present, Your Honor. Will the jury please stand. *(Jury, which has previously been selected but which knows nothing about the case, stands for an instant, then sits.)*

JUDGE. Mr. Clerk, we shall now hear the first case.

PLAINTIFF *(dressed in Russian smock)*. Your Honor, I am a poor farmer. My home is outside Vladivostok near the river. All my life I have tilled the soil in that place, raking out stones and planting
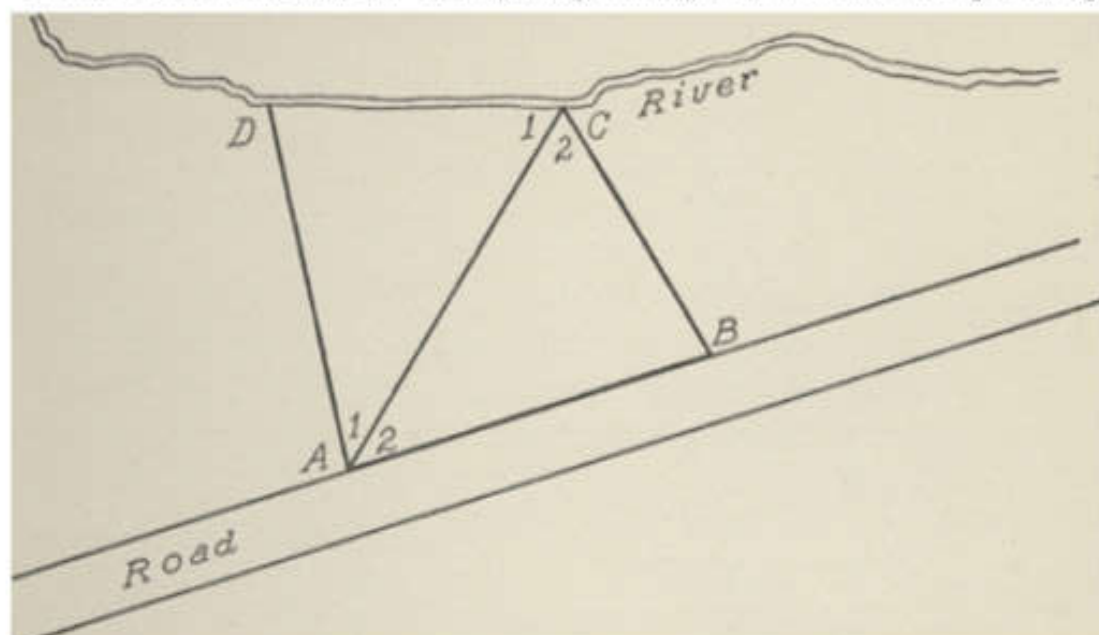
wheat. But there were many stones and the wheat has not been full and strong. I have worked very hard and I am poor. I sell my wheat for only a few kopecks, hardly enough to support my family. And now there comes this man, who says he must buy half my land. And if I will not sell it, he must buy it anyway. The government will take it from me. He says he will pay. All right, I am willing. But he puts two stakes in the ground and ties a rope between and says: On this side, your land; here, mine. And I know he takes more than half my land. True, he gives me the land by the river but he gives me none by the road. I wouldn't mind if I didn't think he takes the lion's share. And when I object, he sits on a rock and draws pictures of things which he calls angles and explains until my head is weary. I am sure that I understand about his angles, but he will not measure all around the fields.

DEFENDANT. No, no, I will not measure all around the fields. When

I know the length of two sides of a field of this sort, and the size of the angle between those sides, then I know whether or not one field is as big as another. Now, Your Honor, the farm looks like this. It goes from A to B, to C, to D and back to A again. *(He produces a map.)*

The side CB is as long as the side DC.

PLAINTIFF. How long?

JUDGE *(rapping for order)*. Order! My man, you must not interrupt the court in this way.

PLAINTIFF. But, Your Honor, I would like to know how long that boundary is.

DEFENDANT. I'll tell you. Both CB and DC are 195 meters. But the number doesn't matter. What matters is that their lengths are the same. Also angle C_1 and angle C_2 are equal.

PLAINTIFF. I object, Your Honor. How does this lawyer know the sizes of these boundary lines and angles?

JUDGE. Objection sustained. *(To lawyer)* Please explain.

DEFENDANT. I procured my information from the Registry of Deeds of Vladivostok. In that office are recorded the official measurements of all farms in this neighborhood. Now, the rope that is strung from C to A will be one length whether we consider it the boundary of my field or of his. Then behold! the field I leave him, ADC, is as big as the field I take, ABC.

PLAINTIFF. If he would measure every boundary line and every corner angle, I should be content. But he will not. And so I come to you, honorable sir, for justice.

DEFENDANT. I can show you that it is enough to know the parts which I have described. Nowhere in all the world would an educated man do the unnecessary work which this poor farmer asks. Twenty-two hundred years ago in ancient Greece men saw that it was sufficient to know three parts of a triangle to know its size. Now these lots are triangular in shape, Your Honor, and if a man has studied just a little bit of Euclid, he knows that a triangle has six parts that can be measured—three angles and three sides—and he knows that if he chooses wisely, information about three of the six parts is enough. I will show you that what I say is true. Take two triangles such as these. Can you fancy that triangle ABC be folded over on the line AC, so that it lies somewhere on the other field? Is not the very best way for comparing the sizes of surfaces to lay them one on the other

to see if they fit? If this farmer cannot imagine my field laid over upon his, let him think of a great stiff canvas exactly the shape and size of my field spread out on his. If we place this canvas with the side AC of my field lying upon the side AC of his and let the rest of it go wherever it will, we can examine the edges and find out whether or not the canvas copy of triangle ABC fits on triangle ACD. In this diagram I shall indicate the canvas by a red line.

Your Honor, I have found in the records that angles C_1 and C_2 are equal. If angle C_2 were bigger than angle C_1, what direction would CB take? It would go off here to the north crossing the river. (*Shows it.*)

If angle C_2 were smaller than C_1, where would CB go? It would start from C to run off this way toward the south, and lie somewhere on this side of the river. (*Shows it.*) But the opening of angle C_2 is exactly the same as the opening of angle C_1. Then must not CB take the direction of CD? (*Lawyer draws a short red line along CD starting from C and going only part of the way to D.*) If you are now convinced that the canvas replica of my field when placed on this farmer's field would have the edge CB run along the edge CD, the next thing to find out is how far it runs. If my field is bigger than field ACD, should we not expect the corner B to be farther along the river than D? But it isn't farther along than D. B is exactly where D is, for after I found that CB ran along CD I examined both lines, CB and CD, in the recorded deed and found them of equal length. Don't you see then that the point B must lie on D?

So far, you agree that triangle ABC fits on triangle ACD throughout the length of CA, around the corner C, and throughout the length of CD? Then there is no more work to do. Anyone knows that if a stake is placed at D (where B now is) and another at A, that only one straight line can be drawn between them. And mercifully, both boundary lines BA and DA are straight. Therefore, the straight boundary line DA stretched between D and A must be exactly where the straight line BA is, for it, too, is stretched from D (where B now is) to A.

Honorable Sir, I have examined only three parts of each triangle, two sides and the included angle in each case. In triangle ABC, I examined the lengths of AC and CB and the size of angle C_2, but I did not bother to find out the values of angles B or A_2 or of line BA. In triangle ACD, likewise, I examined only the lengths of sides AC and CD and the size of angle C_1.

But I have shown you that even without knowing the measurements of the other parts, that the first triangle must fit on the second. Would it not be wasting my valuable time to study six parts when I wish to compare the sizes of two triangles? I know that triangles will fit exactly, or, as we say, coincide, when I know that two sides and the included angle of one are equal respectively to two sides and the included angle of the other. I rest my case.

JUDGE *(addressing jury)*. Gentlemen of the jury, I submit the case to you. If you find that the farmer has been given less than his fair share of the land, then bring in your verdict in his favor. But if you find that the government agent has shown that these two triangular lots are of equal size, then you must bring in a verdict for the government. Will you please retire to the adjoining room and decide upon your verdict and report as soon as possible.

(Recess while jury is out)
(Jury returns and reports verdict)

JUDGE. The court is now adjourned.

Junior High School Conference

The Eighth Annual Junior-High-School Conference will be held at New York University on March 18-19, 1932. The central theme of this conference will be: "Improving Junior-High-School Instruction." Two general sessions coming Friday evening and Saturday morning will be followed by some thirty Round Tables related to the central topic.

This conference is unique in that it is a co-operative arrangement, the directive force being vested in an Advisory Committee of thirty-six representative educators distributed throughout the eastern states. The committee just referred to gives practical direction to the character of the conference. New York University offers the facilities for making this clearing house treatment of junior high school problems possible.

The regional character of the conference makes it one of the most important of its kind in the country. In 1931, upwards of 2,000 attended it. The following states took an active part in the conference as revealed by talent and individual attendance: New York, Pennsylvania, New Jersey, Connecticut, Delaware, Maryland, Rhode Island, Massachusetts, and Ohio. It is also interesting to note that the conference has had a remarkable growth. With seven round tables and thirty speakers in 1925, it has expanded to include thirty-one round tables and one hundred and thirty-four speakers for 1931.